Fuzzy Multi-Objective Optimization for the Assignment Problem in Textile Rotary Printing Processes

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Abstract. We propose a fuzzy multi-objective decision model to optimize the assignment problem in the textile rotary printing industry where there are two conflict fuzzy objectives to optimize. On the one hand, it is necessary to minimize the long times it takes to change cylinders in rotary printers. On the other hand, minimize waiting times for customer orders since they arrive until their supply is necessary. The aspiration levels of the both objective functions are considered fuzzy. In order to solve this problem, we apply a fuzzy goal programming approach based on a satisfying method. Finally, we present an example based on a real-word industrial problem.

Keywords. Fuzzy optimization, multi-objective, assignment, production planning, textile industry.

1 Introduction

Traditionally, assignment or load problems in production planning focus on the allocation of orders to work centers by indicating which operations will carry out each one. In a general assignment problem, there can be several alternative work centers to do the operations. If they all have the same efficiency, operations can be assigned by keeping a homogeneous load in all the work centers or by subsequently saturating work centers. If the work centers have different efficiencies, it is desirable to assign each operation with a greater efficiency to the work center. The typical objective of the assignment problem is to minimize the time or cost involved in processing orders. Thus allocation of orders to work centers is done by minimizing the total time or cost spent.

The article by Kuhn (1955) was seminal on the solution of the classic assignment problem. Several works like Burkard (2002), Öncan (2007) and Pentico (2007), among others, have reviewed and identified the development and applications of the assignment problem. Heuristic and metaheuristic (Lalla-Ruiz et al. 2016) algorithms and mathematical programming (Azab, 2016) models are generally proposed to address the assignment problem.

We herein propose a fuzzy multi-objective decision model, which considers the fuzzy trade-off between the setup times and waiting times for the orders to be processed. Then we apply a fuzzy goal programming (FGP) approach to solve the assignment problem of production orders in a textile rotary printing company. We also present an interactive solution methodology to convert this FGP model into an auxiliary crisp single-objective integer model and to find a preferred compromise solution interactively. For illustration purposes, an example based on a real-world industrial problem is presented. Other goal programming approaches for the assignment problem in production planning contexts can be found in Jahanshahloo and Afzalinejad (2008) and Mehlawat and Kumar (2014), although they are oriented more to the general assignment problem solution rather than to its application as in this paper. It is important to highlight the work by Gupta and Mehlawat (2014), which proposes a new possibilistic programming approach to solve a fuzzy multi-objective assignment problem. However in this case, the objective function coefficients are characterized by triangular possibility distributions.

The rest of the paper has been organized as follows: Section 2 presents the description of the problem. Section 3 formulates the multi-objective model for the assignment problem. Section 3 presents a solution methodology. Section 4 develops an application example based on real-world data from a textile industry. Finally, Section 5 provides the conclusions and further research.

2 Problem Description

The problem addressed in this paper is based on the textile rotary printing industry. These textile rotary printing companies usually manage different work processes (washing train, whitening train, rotary printing, jigguers, finishing, etc.), but have a main or bottleneck process, rotary printing. This technology, which is actually being transformed into digital printing, implies the optimal assignment of the orders to the required in rotary printing processes.

This is a service company, i.e. customers provide the raw material, the fabrics to be printed, and the company stamps these fabrics. Obviously the machinery and process are complex because each model has its own cylinders to stamp, which implies continuous setup times.

The problem consists in the optimal assignation of the customer orders to each rotary printing machine by minimizing setup times and the time that orders take to be processed. Furthermore, the following assumptions were considered.

- Each order can be assigned to only one rotary machine
- There is a limited resources capacity
- All production must equal customer demand
- An order cannot be processed in resource j at least one this order has been assigned to it
- There are orders-resources pairs whose assignation to the machine is not possible due to fabric width requirements
- There are orders-resources pairs whose assignation to the machine is not possible due to the drawing type

- There orders-resources pairs whose assignation assignation to the machine is prioritized due to customer characteristics.
- There are orders-resources pairs whose assignation assignation to the machine is prioritized due to fabric characteristics.
- It is not possible to stamp more than a quantity of meters of some specific textile model by period *t*

3 Model Formulation

The multi-objective model for the assignment problem in textile rotary printing processes is formulated as follows:

Sets of indices

J	Set of rotary printers (j = 1, …, J)
T	Set of periods (t = 1, …T)
P	Set of order lines (p = 1, …, P)
PBC	Set of order lines from the main client (pbc = 1,…, PBC)
$PRIOR$	Set of urgent orders ($prior$ = 1, …, $PRIOR$)
IMP	Set of products of a special size (imp = 1, …, IMP)
IZI	Set of products that are mandatory to stamp in a specific rotary printer (izi = 1, …, IZI)

Parameters

Cil_p	Number of cylinders needed to stamp reference/order line p
$Ccil_j$	Time needed to change a cylinder in rotary printer j
PR_j	Production rate in rotary printer j
CAP_j	Available capacity production in rotary printer j
d_p	Amount of meters demanded per order each order line p
$days_p$	Waiting days of order line p
pde_p	Special orders that belong to a specific fabric type
MDe	Maximum amount of meters of a specific fabric type to stamp during a time period

Objective functions

The first objective function minimizes the fuzzy total time for changing cylinders in rotary printers.

$$\text{Min } z_1 \cong \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{t=1}^{T} Cil_p \cdot Ccil_j \cdot Y_{pjt} \tag{1}$$

Also, the following objective function minimizes the fuzzy total waiting time for orders.

$$\text{Min } z_2 \cong \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{t=1}^{T} days_p \cdot t \cdot Y_{pjt} \tag{2}$$

Constraints

Constraint (3) limits that each order could be assigned only to one rotary printer.

$$\sum_{j=1}^{J}\sum_{t=1}^{T} Y_{pjt} = 1 \qquad \forall p \tag{3}$$

Constraint (4) establishes the available capacity in each rotary printer.

$$\sum_{p=1}^{p} PR_j \cdot d_p \cdot Y_{pjt} + \sum_{p=1}^{p} Cil_p \cdot Ccil_j \cdot Y_{pjt} \leq CAP_j \qquad \forall j \tag{4}$$

Constraints (5) and (6) avoid assignments of orders that cannot be performed in a specific rotary printer.

$$Y_{(p=imp),(j=stork),t} = 0 \qquad \forall t \tag{5}$$

$$Y_{(p=izi),(j=stork),t} = 0 \qquad \forall t \tag{6}$$

Constraint (7) establishes assignments for orders with higher priority.

$$\sum_{j=1}^{J}\sum_{t=1}^{4} Y_{(p=prior),jt} = 1 \tag{7}$$

Constraint (8) limits the maximum amount of a specific fabric to stamp in a time period *t*.

$$\sum_{p=1}^{P} d_p \cdot Y_{pjt} \cdot pde_p \leq MDe \qquad \forall j,t \tag{8}$$

Constraint (9) establishes the integrity and binary conditions for decision variables.

$$Y_{pjt} \in \mathrm{Z} \text{ and binary} \tag{9}$$

4 Solution Methodology

In order to solve the multi-objective model (1)-(9) presented in the previous section, we propose a fuzzy multi-objective programming approach based on the Fuzzy Sets Theory (Zadeh, 1965; Bellman and Zadeh, 1970). The literature provides different forms of a membership function to represent fuzzy objective functions, such as linear, exponential, hyperbolic, hyperbolic inverse, piece-wise linear, etc. Among them, linear membership functions can generate good quality, efficient and computationally linear models. As the two objectives are of the minimizing type in our case, both membership functions are non increasing:

$$\mu_k = \begin{cases} 1 & z_k < z_k^l \\ \dfrac{z_k^u - z_k}{z_k^u - z_k^l} & z_k^l < z_k < z_k^u \\ 0 & z_k > z_k^u \end{cases} \tag{10}$$

where μ_k is the membership function of z_k , while z_k^l and z_k^u are respectively the lower and upper bounds of the fuzzy aspiration level of z_k . We can determine each membership function by asking the decision maker to specify the fuzzy objective value interval by optimizing each objective function separately.

In this section, a satisfying method for FGP, with the different importances and priorities proposed by Cheng (2013), is applied in order to obtain a set of solutions to the proposed model. This method takes into account both the fuzziness and preemptive priorities of the goals by maximizing the sum of achievement degrees (μ_k) and the sum of importance difference between objectives. The decision maker can adjust the relative importance relations among the objectives by increasing or decreasing λ.

$$\begin{aligned}
& Max \sum_{i=1}^{l} \mu_i + \lambda \sum_{i=1}^{l} \sum_{j=1}^{l} b_{ij}\delta_{ij} \\
& s.t. \\
& \mu_i \leq 1 \\
& b_{ij} = \begin{cases} 1 \; if \;\; z_i \in S_{r'} \; and \; z_j \in S_{r'+1}, r' = 1,\ldots,k-1 \\ \qquad 0 \qquad otherwise \end{cases} \\
& \delta_{ij} = \mu_i - \mu_j \;\; for \; all \; b_{ij} = 1 \\
& S_r = \{ z_j \mid z_j \; belongs \; to \; the \; rth \; priority \; level, i = 1,\ldots,l \}, r = 1,\ldots,k \\
& S_{r1} \cap S_{r2} = \varnothing, if \;\; r1 \neq r2, r1 \; and \; r2 \in \{1,\ldots,k\} \\
& \lambda \geq 0 \\
& x \in G(x) \in R^n
\end{aligned} \tag{11}$$

where $x = (x_1, x_2, \ldots, x_n)$ represents the decision vector and and $G(x)$ includes the system constraints.

5 Application

Here we present an application example provided by a Spanish SME that belongs to textile industry to validate and evaluate the results of our proposal.

The proposed model was implemented in the MPL language, V4.2 The resolution was carried out with the Gurobi 5.6.2 solver. The input data and model solution values were processed with Microsoft Access database 2010.

The following assumptions and input data are considered. The study considers two available rotary printers of two different classes, known as STORK (automatic printer machine with 12 cylinders, 2800 mm width, with a natural gas drying chamber) and ZIMMER (automatic printer machine with 12 cylinders, 576 cm. length table and 3200 mm width, a natural gas drying chamber). A half month planning horizon and daily period planning were considered, as were 212 orders to be assigned. Of these, 128 orders belong to the most important customer, 29 orders have high priority, 102 orders are of a special size and 1 order has to be mandatorily processed in the ZIMMER rotary printer due to fabric characteristics. The number of cylinders per order fluctuates between 1 and 13 cylinders. Four 4 minutes are needed to change each cylinder in the STORK printer and 10 minutes in the ZIMMER printer. The average production rates are identical for each rotary printer (0.033 meters/minute). The available production capacity for each rotary printer is 24 h. The meters demanded per order range between 120 and 10689 m. Waiting days per order fluctuate between 1 and 47 days.

Forty-five special orders correspond to the specific devore fabric. The maximum amount of meters of this fabric to be stamped during a period is set at 2000 meters. Two priority levels are considered: Priority level 1: total time for changing cylinders in rotary printers, (z_1); and Priority level 2: total waiting time for orders (z_2). It is important to highlight that the current procedure for the assignment problem is carried out manually by using a spreadsheet to which different filters are continuously applied. Obviously, this manual procedure is very time-consuming and the results are far reaching from an optimal solution because the complete horizon to consider is difficult to manage at one time when contemplating different filters.

According to the priority levels for each objective function defined previously, the equation (11) can be formulated as follows:

$$
\begin{aligned}
&Max \sum_{i=1}^{2} \mu_i + \lambda \sum_{i=1}^{2} \sum_{j=1}^{2} b_{ij} \delta_{ij} \\
&s.t. \\
&the\ membership\ functions\ (\mu_i, i = 1, 2)\ defined\ in\ Eq. (10) \\
&b_{ij} = \begin{cases} 1\ if\ z_i \in S_{r'}\ and\ z_j \in S_{r'+1},\ r' = 1 \\ 0 \qquad otherwise \end{cases} \\
&\delta_{ij} = \mu_i - \mu_j\ for\ all\ b_{ij} = 1 \\
&S_1 = \{z_1\}, S_2 = \{z_2\} \\
&\lambda \geq 0 \\
&and\ constraints\ (3) - (9)
\end{aligned}
$$

In order to determine the aspiration levels limits for each objective, we have calculated the positive ideal and negative ideal solutions by optimizing separately both objective functions with the corresponding model constraints. Alternatively, these aspiration levels limits could be provided by DM according to his/her criteria.

Table 1. Results.

λ	z_1 (min)	z_2 (days)	μ_{z1}	μ_{z2}	$\sum_i \mu_{z_i}$	$\sum_i \sum_j b_{ij}\delta_{ij}$
0.1	6278	8651	0.9986	0.9089	1.9075	0.0898
0.2	6278	8756	0.9986	0.9048	1.9034	0.0939
0.3	6278	8756	0.9986	0.9048	1.9034	0.0939
0.4	6272	9004	1.0000	0.8950	1.8950	0.1050
0.5	6272	9004	1.0000	0.8950	1.8950	0.1050
0.6	6272	9004	1.0000	0.8950	1.8950	0.1050
0.7	6272	9000	1.0000	0.8952	1.8952	0.1048
0.8	6272	9027	1.0000	0.8941	1.8941	0.1059
0.9	6272	9516	1.0000	0.8749	1.8749	0.1251
1.0	6272	14835	1.0000	0.6659	1.6659	0.3341
1.1	6272	29698	1.0000	0.0820	1.0820	0.9180
1.2	6272	29715	1.0000	0.0814	1.0814	0.9186
1.3	6272	29735	1.0000	0.0806	1.0806	0.9194
1.4	6272	29735	1.0000	0.0806	1.0806	0.9194
1.5	6272	30303	1.0000	0.0583	1.0583	0.9417

From Table 1, we can see that the total time for changing cylinders in rotary printers, z_1, is 6.278 minutes and the total waiting time for orders, z_2, is 8.652 days for a λ = 0.1. It is observed that for a value λ = 1.5 the aspiration level of z_2 is lower than 6%. However, this is a representative set of solutions and the DM could extend it until the solution would remain invariable. Additionally it can be observed that the value z_1 is practically constant and, therefore, is the objective z_2 which determines the solution quality. In this sense, it could be useful for DM to explore the interdependency between objectives through the determination of the Pareto frontier.

This proposed FGP model provides a better set of solutions in time terms to for change the cylinders in rotary printers for orders than the manual procedure currently followed in the firm under study (7254 minutes). On the contrary, the proposed model only improves the value for this more prioritized objective function although the z_2 value achieves a 90.89% of aspiration level, 8651, in comparison to manual procedure which obtains a total waiting time of 6820 days.

6 Conclusions

This study has addressed the assignment problem in the textile rotary printing industry, which is currently and mainly solved manually or with spreadsheets.

In order to obtain optimal results, a fuzzy multiple-objective mathematical programming model has been developed. For the purpose of solving this fuzzy multi-objective model, we propose a solution methodology based on FGP. The FGP approach, originally proposed by Cheng (2013), consists in a satisfying method which takes into account both the fuzziness and preemptive priorities of the goals.

This proposal has been applied to a real-world problem in a Spanish SME that belongs to textile rotary printing industry. Compared to its current manual procedure, this FGP model offers better performance in terms of minimization of setup cylinder and computational times.

The advantages of this proposal are related to: modeling and establishing priorities for the assignment problem, which are traditionally measured through the costs estimated with difficulty by companies; and applying and validating a solution methodology for solving fuzzy multi-objective models based on FGP in a real-world problem.

Regarding the limitations of this work, we describe them through further research proposals: (i) using metaheuristics to improve the efficiency of the solution methodology for large-scale problems is a forthcoming work; (ii) possible difficulty in implementing the proposed model in real environments because industrial practitioners look for tools whose general purpose is to solve production problems easily without having to learn new modeling or programming languages; and (iii) developing a decision support system to systematize model configuration and running by integrating it into the firm's current information system.

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